IJRSET Volume 2, Issue 5 Pages: 7-11

ISSN 2394-739X

International Journal for Research in Science Engineering and Technology

p-mean Approximation for HPWL

[1] **B N Bhramar RAY,**
[1] Department of Computer Science & Applications,
[1] Utkal University, Vani Vihar.

Abstract:-

In VLSI analytical placement, Half-perimeter wirelength (HPWL) is used as objective function. Inspired by logarithm-sum-expontial (LSE) wirelength model, in this paper we have introduced a smooth function for HPWL and studied its convergence properties, derived error bound and numerical stability.

We also compare its runtime with widely used LSE and recently proposed weighted average(WA) [3]and (γ,p)[8] wirelength models. The runtime of the model is smaller than LSE and ABS model but comparable with WA model.

Keywords: - wirelength; VLSI; hypergraph; placement;

1. INTRODUCTION

In physical design of VLSI, placement is a challenging still today. Here cells physical locations are optimally determined by analytical placement techniques framing the placement problem as an optimization problem with constraints such as congestion, routing delay, power etc the objective function for placement is HPWL and need to be smooth and convex. Recent widely used analytical placers are Aplace [5], mPL6 [2], FastPlace [10], NTUPlacer [1], Kraftwerk [9] and SimPL [6]. Analytical placers like Aplace [5], mPL6 [2], NTUPlacer [1] use HPWL as their only objective due its simplicity and ease of calculation.

Various smooth functions for HPWL are proposed in the literature such as [5], [7], [3] and [8]. which are often used by analytical placers stated above.

LSE and WA models are very popular and in analytical placement in this work we propose an iterative smooth approximation to HPWL function which can be applied to analytical placement.

A. Contributions

Our contributions in this paper are listed below.

1) We have proposed a smooth approximation to the max function. Using the smooth max function, we derive an iterative smooth wirelength function for half-perimeter wirelength model.

2) We study the convexity, convergence properties and derive an upper bound of errors of max function.

3) We also discuss about implementation issues, numerical stability of the proposed wirelength model and runtime for two variable max function for various models such as :

LSE model, absolute wirelength model (ABSWL) [7], WA model and (γ, p) models.

The remainder of this paper is organized as follows.

Section II discusses existing wirelength models. Section III details the new wirelength model, studies its convexity, convergence properties and derives an upper bound of errors. Section IV presents runtime consideration. Finally, the conclusions and future scope of the work are given in Section V.

2. HPWL FORMULATION AND REVIEW OF EXISTING WIRELENGTH MODELS

The circuits of a placement are denoted by a hypergraph H(V, E), where V is the set of fixed or movable blocks or pads, and E is a set of nets. If we denote the bottom left corner of a block in chip by (xi, yi)(1 ≤ i ≤| V |), then the HPWL of a net e is given by

$$HPWL_e = \max_{i\in e}\{x_i\} - \min_{i\in e}\{x_i\} + \max_{i\in e}\{y_i\} - \min_{i\in e}\{y_i\} \quad \textbf{(1)}$$

Then the total HP W L of a placement is given by sum of the HPWL of all nets.

$$HPWL = \sum_{e\in E} HPWL_e \quad \textbf{(2)}$$

A. Review of Existing HPWL Wirelength Models The wirelength function given by (Eqn(1) and (2)) is hard to minimize due to the presence of max and min functions, as these functions are not differentiable. Analytical placer reformulates HPWL by replacing these functions by their smooth approximations before the placement problem is solved by non linear mathematical programming techniques. There are many smooth approximations for max and min functions. Some of them are discussed below.

1. Logarithm-Sum-Exponential Wirelengt Model(LSE)[11]

For real parameter γ → 0, smooth approximation to HPWL of a net e is given by

$$LSEWL_e = \gamma \ln\Big(\sum_i e^{x_i/\gamma}\Big) + \gamma \ln\Big(\sum_i e^{-x_i/\gamma}\Big) + \gamma \ln\Big(\sum_i e^{y_i/\gamma}\Big) + \gamma \ln\Big(\sum_i e^{-y_i/\gamma}\Big) \quad (3)$$

This is a popular wirelength model for HPWL and is used by analytic placers discussed in [1], [5],[2].

2. Weighted Average Wirelength Model (WAWL)[3]

If x and y coordinates of blocks of a net e are denoted by xe and ye respectively, then the weighted average HPWL of a net is given by

$$WAWL_e = (X_{max}(x_e) - X_{min}(x_e)) + (Y_{max}(y_e) - Y_{min}(y_e)) \quad (4)$$

Where

$$X_{\max}(x_e) = \frac{\sum_{v_i\in e} x_i \exp(x_i/\gamma)}{\sum_{v_i\in e} \exp(x_i/\gamma)}$$

$$X_{\min}(x_e) = \frac{\sum_{v_i\in e} x_i \exp(-x_i/\gamma)}{\sum_{v_i\in e} \exp(-x_i/\gamma)}$$

$$Y_{\max}(y_e) = \frac{\sum_{v_i\in e} y_i \exp(y_i/\gamma)}{\sum_{v_i\in e} \exp(_i/\gamma)}$$

$$Y_{\min}(y_e) = \frac{\sum_{v_i\in e} y_i \exp(-y_i/\gamma)}{\sum_{v_i\in e} \exp(-y_i/\gamma)}$$

and $\gamma \to 0$.

The authors in Theorem 2[3], proved that the errors upper bounds of WAWL model were less than the errors upper bounds of LSE wirelength model.

3. (γ, p)-Wirelength Model[8]

If x and y coordinates of blocks of a net e are denoted by x_e and y_e respectively, then for real parameters γ → 0, p → ∞, (γ, p)-wirelength model of a net e is given by

$$WAWAL = (X^{(\gamma,p)}(x_e) - X^{(-\gamma,-p)}(x_e) + (Y^{(\gamma,p)}(y_e) - Y^{(-\gamma,-p)}(y_e)) \quad (5)$$

Where

$$X^{(\gamma,p)}(x_e) = \frac{\sum_{v_i\in e} x_i^{p} \exp(x_i/\gamma)}{\sum_{v_i\in e} x_i^{p-1} \exp(x_i/\gamma)}$$

$$X^{(-\yen,-p)}(x_e) = \frac{\sum_{v_i \in e} x_i^{-p} \exp(-x_i / \yen)}{\sum_{v_i \in e} x_i^{-p-1} \exp(-x_i / \yen)}$$

$$Y^{(\yen,p)}(y_e) = \frac{\sum_{v_i \in e} y_i^{p} \exp(y_i / \yen)}{\sum_{v_i \in e} y_i^{p-1} \exp(y_i / \yen)}$$

$$Y^{(-\yen,-p)}(y_e) = \frac{\sum_{v_i \in e} y_i^{-p} \exp(-y_i / \yen)}{\sum_{v_i \in e} y_i^{-p-1} \exp(-y_i / \yen)}$$

The authors in Theorem 5[8], proved that the errors upper bounds of (γ, p)-wirelength model were less than the errors upper bounds of WAWL model and LSE wirelength model. Interestingly, (γ, p)-wirelength model reduces to WAWL model, when p = 1.

4.Absolute Wirelength Model(ABSWL)[7] For real parameter β → ∞, an approximation to the two variable max function max(x_1, x_2) is given by

$$\text{ABSBMAX}(x_1,x_2) = \frac{1}{2}(x_1+x_2+|x_1 - x_2|)$$

$$\approx \frac{1}{2}(x_1 + x_2 + \frac{1}{\ }(\ln 2 + \ln(1 + \cosh(\ \ (x_1 - x_2)))))$$

Generalizing ABSBMAX(x_1, x_2) to n variables max function, smooth formulation for HPWL can be obtained. In [7] it is shown through simulation that estimation upper bound error of ABSWL model is less than LESWL model.

3. PROPOSED WIRELENGTH MODEL

In this section, we present an iterative smooth wirelength function for HPWL providing smooth approximations to maximum and minimum functions of equations (1) and (2). Let $\underline{x_e}$ = (x_1, x_2, ...x_n) and y_e = (y_1, y_2, ...y_n) be x and y be coordinates of net e respectively. Without loss of generality assumes these coordinates are positive real constants. Then for real parameters p → +∞ define weighted average of x_e by

$$X^p(x_e) = \frac{\sum_{i=1}^{n} x_i^p}{\sum_{i=1}^{n} x_i^{p-1}} \tag{8}$$

We call $X^p(x_e)$ as p-mean of x_e.

A. Convergence Properties

Since $X^p(x_e)$ is twice differentiable for $x_i \in x_e$, and the derivative is positive(i.e. the Hessian matrix is positive definite), We have the following Lemma 1.

Lemma 1. $X^p(x_e)$ is strictly convex and continuously differentiable functions of x_e.

Theorem 1. If x_{min} and x_{max} are minimum and maximum of x_1, x_2, ...x_n. Then we have

(i) $\lim_{p \to +\infty} X^p(x_e) = x_{max}$

(ii) $\lim_{p \to -\infty} X^p(x_e) = x_{min}$

Proof: Without loss of generality let us assume $x_1 \geq x_2 \geq \ldots \geq x_n$. Now we have

$$X^p(x_e) = x_1 \frac{1 + \sum_{i=2}^{n} (x_i / x_1)^p}{1 + \sum_{i=2}^{n} (x_i / x_1)^{p-1}}$$

Making p → +∞ Theorem 1(i) follows. Similarly Theorem 1(ii) follows.

Using p-mean smooth approximation to HPWL model is given by

$$\sum_{e \in E} (X^p(x_e) - X^{-p}(x_e) + Y^p(y_e) - Y^{-p}(y_e))$$

Where $Y^p(y_e)$ is the p-mean of y_e and $X^{-p}(x_e)$, $Y^{-p}(y_e)$ are corresponding approximations for x_{min} and y_{min} respectively.

Let $ErrX^p(x_e)$ be estimation error of p-mean of x coordinates of net e respectively. Then we have the following upper bounds error.

Theorem 2: $0 \leq ErrX^p(x_e) \leq$

$$\frac{x}{1 + ((x_{max} / x_{min})^{p-1}) / n}$$

where $\Delta x = x_{max} - x_{min}$.

Proof : Let us assume $x_1 \geq x_2 \geq \ldots \geq x_n$ and denote

$\Delta x_i = (x_1 - x_i)$. Now the error expression for maximum function for net e by p-mean is given by

$$ErrX^p(x_e) = x_1 - X^p(x_e) = \frac{\sum_{i=2}^{n} x_i^{p-1}(x_1 - x_i)}{\sum_{i=1}^{n} x_i^{p-1}} \quad (9)$$

Inorder to get an upper bound of error, let us differentiate equation (9) partially with respect to x_i for $(2 \leq i \leq n)$ and make them equal to 0's. That is for any i, $\partial\, ErrX^{p^*}(x_e)/\partial x_i = 0$, implies

$$\frac{\sum_{i=1}^{n} x_i^{p-1}}{\sum_{i=2}^{n} x_i^{p-1}(x_1 - x_i)} = \frac{(p-1)}{-x_i + (x_1 - x_i)(p-1)} \quad (10)$$

Now solving the system of equations (10) for $2 \leq i \leq n$, one can conclude that error is maximum when $x_2 = x_3 = ... = x_n$. Using $x_1 = x_{max}$, $x_2 = x_3 = ... = x_n = x_{min}$, from equation (9)

we have $ErrX^p(x_e) = \frac{x(n-1)x_{min}^{p-1}}{x_{max}^{p-1} + (n-1)x_{mix}^{p-1}}$

$$= \frac{x}{1 + ((x_{max}/x_{min})^{p-1})/(n-1)} \leq \frac{x}{1 + ((x_{max}/x_{min})^{p-1})/n} \quad (11)$$

where $\Delta x = x_{max} - x_{min}$. Similarly we can have the same bounds of error for minimum function $ErrX^{-p^*}(x_e)$. From the definitions of maximum and minimum functions(See Theorem 1), we have $ErrX^{p^*}(x_e) \leq x_{max}$ and $x_{min} \leq ErrX^{-p^*}(x_e)$. This implies $ErrXp(xe) \geq 0$. Hence Theorem 2 follows. We have used same line of proof adopted for proving Theorem 1 shown in [3].

4. IMPLEMENTATION AND RESULTS

In this section we shall discuss the choice of parameter p of p-mean function, which will keep the implementation numerically stable. Then we compare the runtimes of two variables wirelength models.

A. Choice of p

If datatype double is used to represent wirelength, then the largest value double can take is $1.797E * 10^{308} \approx e^{710}$. Since for p-mean, x^p can not exceed this value, the largest value that p can take is $p \leq 710/\ln x$. Though in theory p is to be taken large, in practice, one need to scale down the chip dimension W and H sufficiently so that the implementation remain stable. To illustrate the effect of p on wirelength model, we choose ibm01 from ISPD 2004 fixed die benchmark suite. Using 1550 × 1530 grids we place the circuit using NTUPlacer[1]. Then we measure the half perimeter wirelegth using exact calculations. Without scaling the chip dimension the largest value that p can take is 710 ln 1530. We choose p slightly larger than this value . Then we steadily increase the value of p and simultaneously scale down the chip dimensions. The effect of larger p on errors for this calculation is shown in Table I. From table, it is evident that the errors go down steadily as p increases.

p	10	20	40	80	100
Error % in p-mean	32.27	20.72	12.2	6.5	5.28

TABLE 1: Effect of p on Approximation

B. Runtime Consideration

As 70% nets in most circuits are two terminal nets, for these nets, analytical placer calculates two-variable approximations to the max function several times during its local search.

Like [7] we also compare the runtimes for two-variable maximum function for LSE, ABS, WA, (γ, p)-mean and p-mean wirelegth models. For this we generated 60×10^6 pairs of random real numbers and passed them as arguments to these two-variables maximum functions. The averaged runtimes over several experiments are listed in Table II. From table one can see p-mean and WA wirelngth models have least runtimes.

Method	LSE	WAWL	ABSWL	(γ,p)-mean	p-mean
Runtime (s)	16.8	11.3	17.9	19.1	11.9

TABLE 2: Runtime of 2-Variable Approximations

CONCLUSIONS AND FUTURE WORK

We proposed an iterative wirelength model for HPWL function and studies its convexity, convergence properties and compared its two variable function run time with existing models. The runtime of the proposed model is less than LSE, ABSWL and (γ,p) models. In future we aim to study the effect on placement.

REFERENCES

[1] T. chieh Chen, Z. wei Jiang, T. chang Hsu, H. chen Chen, and Y. wen Chang. A high-quality mixed-size analytical placer onsidering preplaced blocks and density constraints. In ICCAD, pages 187–192, 2006.

[2] J. Cong and G. Luo. Highly efficient gradient computation for densityconstrained analytical placement methods. In ISPD, pages 39–46, New York, NY, USA, 2008. ACM.

[3] M.-K. Hsu, Y.-W. Chang, and V. alabanov. Tsv-aware analytical placement for 3d ic designs. In Proceedings of the 48th Design Automation Conference, DAC '11, pages 664–669, New York, NY, USA,2011. ACM.

[4] A. B. Kahng and S. Reda. A tale of two nets: studies of wirelength progression in physical design. In SLIP, pages 17–24, 2006.

[5] A. B. Kahng, S. Reda, and Q. Wang. Architecture and details of a high quality, large-scale analytical placer. In In Proc. ICCAD, pages 890–897,2005.

[6] M.-C. Kim, D. Lee, and I. L. Markov. Simpl: an algorithm for placing vlsi circuits. Commun. ACM, 56(6):105–113, 2013.

[7] B. Ray and S. Balachandran. A new wirelength model for analytical placement. VLSI, IEEE Computer Society Annual Symposium on, 0:90–95, 2011.

[8] B. N. B. Ray and S. Balachandran. An efficient wirelength model for analytical placement. In DATE, pages 1711–1714, 2013.

[9] P. Spindler, U. Schlichtmann, and F. M. Johannes. Kraftwerk2 – a fast force-directed quadratic placement approach using an accurate net model. IEEE Trans. on CAD of Integrated Circuits and Systems,27(8):1398–1411, 2008.

[10] N. Viswanathan and C. C.-N. Chu. Fastplace: efficient analytical placement using cell shifting, iterative local refinement and a hybrid net model. In ISPD, pages 26–33, New York, NY, USA, 2004. ACM.

[11] W.C.Naylor, R.Donelly, and L.Sha. Non-linear optimization system and method for wirelength and delay optimization for an automatic electric circuit placer. In US patent 6,301,693, 2001.