



ON (1, 2)-GENERALIZED SEMI-PRE-REGULAR – CLOSED SETS IN BITOPOLOGICAL SPACES

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ABSTRACT- This dissertation entitled “on (1,2)-generalized semi-pre-regular closed sets in bitopological spaces” discuss the concepts of (1,2)-generalized pre-regular closed sets, (1,2)-generalized pre-regular open sets, (1,2)-generalized pre-regular continuous functions and (1,2)-generalized pre-regular irresolute functions and their abstract properties in bitopological spaces. Finally (1,2)-generalized semi-pre-regular closed sets in defined and we have discussed some of its properties.

Keyword- [Generalized Semi-Pre-Regular, Closed Sets, Bitopological Spaces]

(1, 2)- GENERALIZED SEMI-PRE-REGULAR –CLOSED SETS: DEFINITION:

A subset A of (X, τ_1, τ_2) is called (1,2)-generalized semi-pre-regular closed (briefly (1,2)-gspr-closed) if $(1,2)\text{-spcl}(A) \subset U$, whenever $A \subset U$ and $U \in (1,2)\text{-RO}(X)$. The family of all (1,2)-gspr-closed sets of X is denoted by $(1,2)\text{-GSPRCL}(X)$.

1. THEOREM

- i) Every (1,2)closed set is (1,2)-gspr-closed set.
- ii) Every (1,2)-rg-closed set is (1,2)-gspr-closed set.
- iii) Every (1,2)-gs-closed set is (1,2)-gspr-closed set.
- iv) Every (1,2)-sg-closed set is (1,2)-gspr-closed set.
- v) Every (1,2) -closed set is (1,2)-gspr-closed set.
- vi) Every (1,2)-pg-closed set is (1,2)-gspr-closed set.
- vii) Every (1,2)-gp-closed set is (1,2)-gspr-closed set.
- viii) Every (1,2)- g-closed set is (1,2)-gspr-closed set.
- ix) Every (1,2)- -closed set is (1,2)-gspr-closed set.

PROOF:

i) Let A be a (1,2)closed set.

To Prove: A is (1,2)-gspr-closed set.

Suppose U is a (1,2)-regular open set, such that $A \subset U$.

Since A is a (1,2)closed set, $(1,2)\text{cl}(A) = A \subset U$.

$\therefore (1,2)\text{-spcl}(A) \subset (1,2)\text{cl}(A) \subset U$.

(ie) $(1,2)\text{-spcl}(A) \subset U$.

Thus, A is a $(1,2)$ -gspr-closed set.

ii) Assume that A is a $(1,2)$ -rg-closed set.

Let U be a $(1,2)$ -regular open set, such that $A \subset U$.

Since A is a $(1,2)$ -rg-closed set, $(1,2)\text{-cl}(A) \subset U$.

$\therefore (1,2)\text{-spcl}(A) \subset (1,2)\text{-cl}(A) \subset U$.

(ie) $(1,2)\text{-spcl}(A) \subset U$.

Thus, A is a $(1,2)$ -gspr-closed set.

iii) Suppose U is a $(1,2)$ -regular open set, such that $A \subset U$.

Every regular open set is γ -open, U is a $(1,2)$ - γ -open.

Since A is a $(1,2)$ -gs-closed set, $(1,2)\text{-scl}(A) \subset U$.

$\therefore (1,2)\text{-spcl}(A) \subset (1,2)\text{-scl}(A) \subset U$.

(ie) $(1,2)\text{-spcl}(A) \subset U$.

Thus, A is a $(1,2)$ -gspr-closed set.

iv) Assume that A is a $(1,2)$ -sg-closed set.

Let U be a $(1,2)$ -regular open set, such that $A \subset U$.

Since A is a $(1,2)$ -sg-closed set, $(1,2)\text{-scl}(A) \subset U$.

$\therefore (1,2)\text{-spcl}(A) \subset (1,2)\text{-scl}(A) \subset U$.

(ie) $(1,2)\text{-spcl}(A) \subset U$.

Thus, A is a $(1,2)$ -gspr-closed set.

v) Let A be a $(1,2)$ -closed set.

To Prove: A is $(1,2)$ -gspr-closed set.

Suppose U is a $(1,2)$ -regular open set, such that $A \subset U$.

Since A is a $(1,2)$ -closed set, $(1,2)\text{-cl}(A) = A \subset U$.

$\therefore (1,2)\text{-spcl}(A) \subset (1,2)\text{-cl}(A) \subset U$.

(ie) $(1,2)\text{-spcl}(A) \subset U$.

Thus, A is a $(1,2)$ -gspr-closed set.

vi) Assume that A is a $(1,2)$ -pg-closed set.

Let U be a $(1,2)$ -regular open set, such that $A \subset U$.

Every regular open set is pre-open, U is a $(1,2)$ -pre-open.

Since A is a $(1,2)$ -pg-closed set, $(1,2)\text{-pcl}(A) \subset U$.

$\therefore (1,2)\text{-spcl}(A) \subset (1,2)\text{-pcl}(A) \subset U$.

(ie) $(1,2)\text{-spcl}(A) \subset U$.

Thus, A is a $(1,2)$ -gspr-closed set.

vii) Suppose U is a $(1,2)$ -regular open set, such that $A \subset U$.

Every regular open set is γ -open, U is a $(1,2)$ - γ -open.

Since A is a $(1,2)$ -gpclosed set, $(1,2)\text{-pcl}(A) \subset U$.

$\therefore (1,2)\text{-spcl}(A) \subset (1,2)\text{-cl}(A) \subset U$.

(ie) $(1,2)\text{-spcl}(A) \subset U$.

Thus, A is a $(1,2)$ -gspr-closed set.

viii)) Let A be a $(1,2)$ -g-closed set.

To Prove: A is $(1,2)$ -gspr-closed set.

Suppose U is a $(1,2)$ -regular open set, such that $A \subset U$.

Every regular open set is γ -open, U is a $(1,2)$ - γ -open.

Since A is a $(1,2)$ -g-closed set, $(1,2)\text{-cl}(A) \subset U$.

$\therefore (1,2)\text{-spcl}(A) \subset (1,2)\text{-cl}(A) \subset U$

(ie)(1,2)-spcl(A) ⊂ U.

Thus,A is a (1,2)-gspr-closed set.

ix) Assume that A is a (1,2)-semi-pre-closed set.

Let U be a (1, 2)-regular open set,such that A⊂U.

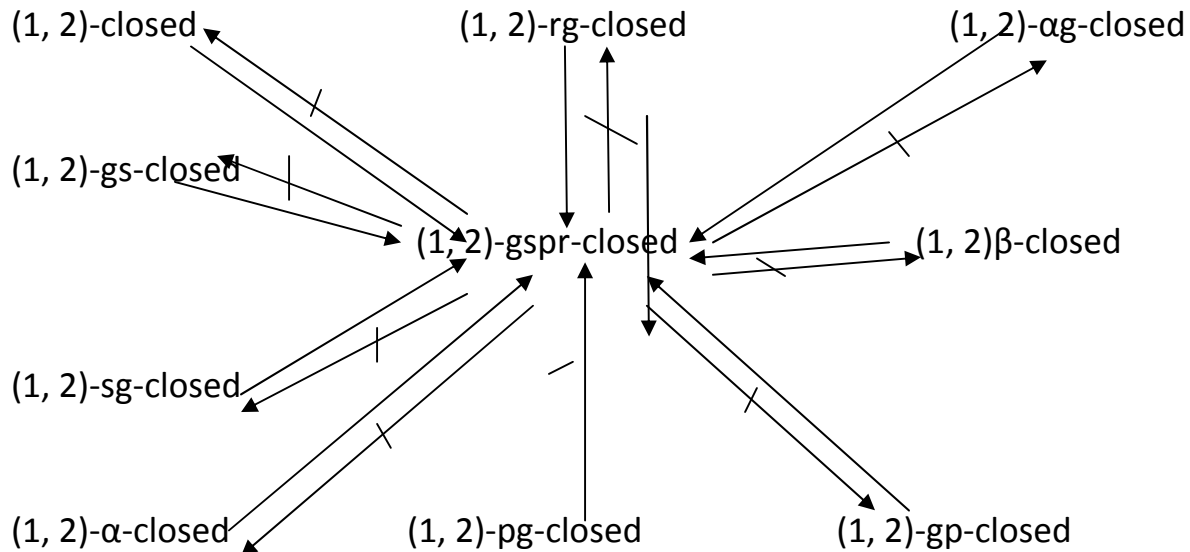
Then(1, 2)- cl(A)=A⊂U.

∴(1,2)- cl(A) ⊂ U.

(ie)(1,2)-spcl(A) ⊂ U.

Thus,A is a (1,2)-gspr-closed set.

ALL THE ABOVE RESULTS CAN BE REPRESENTED BY THE FOLLOWING DIAGRAM:



2. REMARK

Converse of the above theorem need not always be true as shown by the following example.

EXAMPLE:

i)Let X ={a,b,c}

$\tau_1 = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$

Closed sets of X with respect to $\tau_1 = \{X, \emptyset, \{c\}, \{a, c\}, \{b, c\}\}$

$\tau_2 = \{X, \emptyset, \{a\}\}$

Closed sets of X with respect to $\tau_2 = \{X, \emptyset, \{b, c\}\}$

$\tau_1 \tau_2 = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$

Closed sets of X with respect to $\tau_1 \tau_2 = \{X, \emptyset, \{c\}, \{a, c\}, \{b, c\}\}$

To find (1, 2)-regular –open sets in (X, τ_1, τ_2):

A	$\text{int}_{\tau_1}(\text{cl}_{\tau_2}(A))$	$\text{int}_{\tau_2}(\text{cl}_{\tau_1}(A))$	$A = \text{int}_{\tau_1}(\text{cl}_{\tau_2}(A)) = \text{int}_{\tau_2}(\text{cl}_{\tau_1}(A))$
{a}	{a,c}	{a}	Yes
{b}	{b,c}	{b}	Yes
{c}	{c}	\emptyset	No
{a,b}	X	X	No
{a,c}	{a,c}	{a}	No
{b,c}	{b,c}	{b}	No

$\therefore \{X, \emptyset, \{a\}, \{b\}\}$ are (1,2)regular –open sets in (X, τ_1, τ_2) .

To find (1,2)-semi pre-closed sets in (X, τ_1, τ_2) :

A	$\text{int}_{(\tau_1, \tau_2)}(A)$	$\text{cl}_{(\tau_1, \tau_2)}(A)$	$\text{int}_{(\tau_1, \tau_2)}(\text{cl}_{(\tau_1, \tau_2)}(A))$	$\text{cl}_{(\tau_1, \tau_2)}(\text{int}_{(\tau_1, \tau_2)}(A))$
{a}	{a}	{a,c}	{a}	Yes
{b}	{b}	{b,c}	{b}	Yes
{c}	\emptyset	\emptyset	\emptyset	Yes
{a,b}	{a,b}	X	X	No
{a,c}	{a}	{a,c}	{a}	Yes
{b,c}	{b}	{b,c}	{b}	Yes

Thus, $\{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$ are (1,2)-semi pre-closed sets in (X, τ_1, τ_2) .

To find (1,2)gspr-closed sets in (X, τ_1, τ_2) :

Let $A = \{a\}$

$(1,2)\text{-spcl}(a) = \{a\}$

(1,2)-regular-open sets containing {a} are {a}, X.

$\{a\} \subseteq \{a\}, \{a\} \subseteq X$.

Which implies that {a} is (1,2)-gspr-closed set.

Similarly we can prove that $\{\{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ are (1,2)gspr-closed sets in (X, τ_1, τ_2) .

Hence $\{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ are (1,2)gspr-closed sets in (X, τ_1, τ_2) .

(1,2)gspr-closed sets --- (1,2)-closed sets in (X, τ_1, τ_2) :

Let $A = \{a\}$

$(1,2)\text{cl}(\{a\}) = \{a, c\}$

Let $A = \{b\}$

$(1,2)\text{cl}(\{b\}) = \{b, c\}$

Let $A = \{c\}$

$(1,2)\text{cl}(\{c\}) = \{c\}$

Let $A = \{a, b\}$

$(1,2)\text{cl}(\{a, b\}) = X$

Let $A = \{a, c\}$

$(1,2)\text{cl}(\{a, c\}) = \{a, c\}$

Let $A = \{b, c\}$

$(1,2)\text{cl}(\{b, c\}) = \{b, c\}$

Ie., $\{\{a\}, \{b\}, \{a, b\}\}$ are not (1,2)closed but are (1,2)gspr-closed.

Thus, $\{X, \emptyset, \{c\}, \{a, c\}, \{b, c\}\}$ are (1,2)closed sets in (X, τ_1, τ_2) .

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