



Intuitionistic Fuzzy LPP Unbalanced Transportation Problem

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Abstract:-

In this paper, we investigate transportation problem in which cost coefficients are triangular intuitionistic fuzzy numbers. In conventional transportation problem, cost is always certain. This paper develops an approach to solve an intuitionistic fuzzy transportation problem where cost is not deterministic numbers but imprecise ones. Here, the elements of the costs (profits) matrix of the transportation problem are triangular intuitionistic fuzzy numbers. Then its triangular shaped membership and non-membership functions are defined. A new ranking procedure which can be found in [12] and is used to compare the intuitionistic fuzzy numbers so that an Intuitionistic Fuzzy MVAM method may be applied to solve the intuitionistic fuzzy transportation problem. Numerical examples show that an intuitionistic fuzzy ranking method offers an effective tool for handling an intuitionistic fuzzy transportation problem.

Keywords: - Intuitionistic Fuzzy Set, Triangular Fuzzy Number, Triangular Intuitionistic Fuzzy Number, Intuitionistic Fuzzy transportation Problem, Optimal Solution

1. INTRODUCTION

Transportation Problem (TP) is used worldwide in solving real world problems.

Transportation problem plays an important role in an allocate of demand to supply, problems to research teams, etc. The transportation problem a special type of linear programming problem (LPP) in which our objective is to allocate number of sources to n number of destinations (persons) at a minimum cost. To find solution to transportation problems, various algorithms such as linear programming [18, 9, 13, and 17], MVAM algorithm [15], neural network [12], genetic algorithm [8] have been developed.

However, in real life situations, the parameters of allocate problems are imprecise numbers instead of fixed real numbers because time/cost for doing a job by a facility (machine/persion) might vary due to different reasons. The theory of fuzzy set introduced by Zadeh[21] in 1965 has achieved successful applications in various fields. In 1970, Belmann and Zadeh introduce the concepts of fuzzy set theory into the decision- making problems involving uncertainty and imprecision. Amit Kumar et al investigated transportation and Travelling Salesman transportation Problems with cost coefficients as LR fuzzy parameters and Fuzzy linear programming approach for solving fuzzy transportation problems with transshipment [1], Method for solving fully fuzzy transportation problems using triangular fuzzy numbers[3]. In [18], Sathi Mukherjee et al

presented an Application of fuzzy ranking method for solving transportation problems with fuzzy costs. Lin and Wen proposed an efficient algorithm based an labeling method for solving the linear fractional programming case. Y.L.P.Thorani and N.Ravi Sankar did Fuzzy transportation problem with generalized fuzzy numbers.Different kinds of fuzzy transportation problems are solved in the papers [1, 3, 10, 11, 14, and 20].

The concept of Intuitionistic Fuzzy Sets (IFSs) proposed by Atanassov[5] in 1986 is found to be highly useful to deal with vagueness. In [21], Jahir Hussian et all presented An Optimal More-for-Less Solution of Mixed Constrains Intuitionistic Fuzzy Transportation Problems. Here we investigate a more realistic problem, namely intuitionistic fuzzy transportation problem. Let be the intuitionistic fuzzy cost of transportation the j^{th} destination to i^{th} sources. The problem is to find an optimal transportation so that the total intuitionistic fuzzy cost of performing all sources and destination is minimum or the total intuitionistic fuzzy profit is maximum. In this paper, ranking procedure of Annie Varghese and Sunny Kuriakose [2] is used to compare the intuitionistic fuzzy numbers. Finally an Intuitionistic Fuzzy MVAM method may be applied to solve an IFTP.

This paper is organized as follows: Section 2 deals with some basic terminology and ranking of triangular intuitionistic fuzzy numbers, In section 3, provides not only the definition of intuitionistic fuzzy transportation problem but also its mathematical formulation and Fundamental Theorems of an Intuitionistic Fuzzy transportation Problem. Section 4 describes the solution procedure of an intuitionistic fuzzy transportation problem, In section 5, to illustrate the proposed method a numerical example with results and discussion is discussed and followed by the conclusions are given in Section 6.

2. PRELIMINARIES

2.1. Definition: Let A be a classical set, $\delta_A(x)$ be a function from A to [0,1]. A fuzzy set $\tilde{A}_{\alpha T}$ with the membership function is defined by $\tilde{A}_{\alpha T} = \{(x, \delta_A(x)),; x \in A \text{ and } \delta_A(x) \in [0,1]\}$.

2.2. Definition: Intuitionist fuzzy set: Intuitionist fuzzy set (IFS) A in x is a set of ordered triples of universe element, $\tilde{A}_{\alpha IT} = \{(x, \delta_A(x), v_{\bar{A}}(x)); x \in X\}$ and degree of membership to $\delta_A(x)$ degree of non-membership to $v_{\bar{A}}(x)$ are fuzzy numbers with $\delta_A(x) + v_{\bar{A}}(x) \leq 1$ and $\delta_A(x), v_{\bar{A}}(x) \in [0,1]$. When $\delta_A(x) + v_{\bar{A}}(x) = 1$ one obtain the fuzzy set, and if $\delta_A(x) + v_{\bar{A}}(x) < 1$ There is an I=1- $\delta_A(x) - v_{\bar{A}}(x)$.

2.3. Definition: A fuzzy number A is defined to be a triangular fuzzy number in its membership functions $\delta_A(x) : \mathbb{R} \rightarrow [0, 1]$ is equal to A fuzzy number Z is a triangular fuzzy number denoted by a_1, a_2, a_3 , where a_1, a_2, a_3 are real numbers and its membership function is given below: (a_1, a_2, a_3) .

$$\delta_A(z) = \left\{ \begin{array}{l} \frac{z-a_1}{a_2-a_1} \text{ for } a_1 \leq z \leq a_2, \\ \frac{a_2-z}{a_3-a_2} \text{ for } a_2 \leq z \leq a_3 \\ 0 \text{ for otherwise} \end{array} \right\}$$
 Where $a_1 \leq a_2 \leq a_3$.

$\delta_A(z)$	$v_{\bar{A}}(z)$
$0 \text{ for } z < a_1$ $\frac{z-a_1}{a_2-a_1} \text{ for } a_1 \leq z \leq a_2$ $1 \text{ for } z = a_2$ $\frac{a_2-z}{a_3-a_2} \text{ for } a_2 \leq z \leq a_3$ $0 \text{ for } z > a_3$	$1 \text{ for } z < a_1$ $\frac{a_2-z}{a_2-a_1} \text{ for } a_1 \leq z \leq a_2$ $0 \text{ for } z = a_2$ $\frac{a_3-z}{a_3-a_2} \text{ for } a_2 \leq z \leq a_3$ $1 \text{ for } z > a_3$

2.4. Definition: A triangular Fuzzy Number (A is an intuitionistic fuzzy set in R with the following membership function with $\delta_A(z)$ and non membership function $v_{\bar{A}}(z)$)

This TrIFN is denoted by $\tilde{A}_{\alpha IT} = (a_1, a_2, a_3)$ $(a_{1'}, a_{2'}, a_{3'})$ case 1. If $a_{1'} = a_1; a_{3'} = a_3$, then represent triangular intuitionistic fuzzy number, $\tilde{A}_{\alpha T} = (a_1, a_2, a_3)$.

Case 2. If $a_{1'} = a_1, a_2 = a_3 = a_{3'} = m$, then $\tilde{A}_{\alpha IT}$ represent a real number 'm'.

2.5. Definition: $\tilde{A}_{\alpha T}$ and $\tilde{B}_{\alpha T}$ be te two TrIFN. The ranking of \tilde{A}_{IT} and \tilde{B}_{IT} by the R on E, the set of TrIFN is defined as follows.

- (i) $R(\tilde{A}_{IT}) > R(\tilde{B}_{IT})$ iff $\tilde{A}_{IT} > \tilde{B}_{IT}$
- (ii) $R(\tilde{A}_{IT}) < R(\tilde{B}_{IT})$ iff $\tilde{A}_{IT} < \tilde{B}_{IT}$
- (iii) $R(\tilde{A}_{IT}) = R(\tilde{B}_{IT})$ iff $\tilde{A}_{IT} = \tilde{B}_{IT}$
- (iv) $R(\tilde{A}_{IT} + \tilde{B}_{IT}) = R(\tilde{A}_{IT}) + R(\tilde{B}_{IT})$
- (v) $R(\tilde{A}_{IT} - \tilde{B}_{IT}) = R(\tilde{A}_{IT}) - R(\tilde{B}_{IT})$

2.6. Arithmetic Operation: Following are the three operations that can be performed on Triangular fuzzy numbers, Suppose $\tilde{A}_T = (a_1, a_2, a_3)$ and $\tilde{B}_T = (b_1, b_2, b_3)$ are two triangular fuzzy numbers then

- **Addition:** $\tilde{A}_{IT} (+) \tilde{B}_{IT} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
- **Subtraction:** $\tilde{A}_{IT} (-) \tilde{B}_{IT} = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$
- **Multiplication:** $\tilde{A}_{IT} (*) \tilde{B}_{IT} = (t_1, t_2, t_3)$

Where

- $t_1 = \{ a_1 b_1, a_1 b_3, b_1 a_3 \};$
- $t_2 = \{ a_2 b_2, a_2 b_3, a_3 b_2 \};$
- $t_3 = \{ a_3 b_3, a_3 b_1, a_1 b_3 \}$

2.7. Ranking Triangular Intuitionistic Fuzzy Number: The ranking intuitionistic Triangular fuzzy number $\tilde{A}_{\alpha IT} = (a_1, a_2, a_3)$ $(a_{1'}, a_{2'}, a_{3'})$ is defined by $R(\tilde{A}_{IT}) = ((a_1 a_2 a_3) + (a_2 a_3 a_2') + (2(a_2 a_1' a_2') + 2(a_1 a_3 a_2')) / 172$

8 The Ranking technique $R(\tilde{A}_{IT}) < R(\tilde{B}_{IT})$, iff $\tilde{A}_{IT} < \tilde{B}_{IT}$ and $\min(\tilde{A}_{IT}, \tilde{B}_{IT}) = \tilde{A}_{IT}$, $\max(\tilde{A}_{IT}, \tilde{B}_{IT}) = \tilde{B}_{IT}$

2.8. Example: Let $\tilde{A}_{IT} = (1, 2, 3)(8, 9, 10); \tilde{B}_{IT} = (6, 9, 11)(3, 4, 5)$ be any two TrIFN, then its

rank is defined by $R(\tilde{A}_{IT}) = 0.678$, $R(\tilde{B}_{IT}) = 0.9409$ this implies $\tilde{A}_{IT} < \tilde{B}_{IT}$

3. INTUITIONISTIC FUZZY TRANSPORTATION MODEL

Consider the situation of allocate m sources to n destinations and each machine is capable of doing any at different costs. Let $\tilde{C}_{ij} = [\tilde{C}_{ij}^{(1)}, \tilde{C}_{ij}^{(2)}, \tilde{C}_{ij}^{(3)}]$ be an intuitionistic Fuzzy cost of allocate the j^{th} destination to i^{th} sources. Let $\tilde{x}_{ij} = [x^{(1)}, \tilde{x}_{ij}^{(2)}, \tilde{x}_{ij}^{(3)}]$ be the decision variable denoting the transporting of the sources i to the destination j. The objective is to minimize the total intuitionistic fuzzy cost of all the destinations to the available sources at the least total cost. This situation is known as balanced intuitionistic fuzzy transportation problem.

(IFTP) minimize $\delta_A(z) = (\tilde{C}_{ij} \tilde{x}_{ij})$, Min $z =$

$v_{\tilde{A}}(\tilde{C}_{ij}, \tilde{x}_{ij}), \tilde{x}_{ij} \in X$ Subject to $\sum_{j=1}^n [x(1), \tilde{x}_{ij}(2), \tilde{x}_{ij}(3)] = [\tilde{a}_i^{(1)}, \tilde{a}_i^{(2)}, \tilde{a}_i^{(3)}]$ for $i = 1, 2, \dots, m$
 $\sum_{i=1}^m [x(1), \tilde{x}_{ij}(2), \tilde{x}_{ij}(3)] = [b^{(1)}, \tilde{b}_j^{(2)}, \tilde{b}_j^{(3)}]$ for $j = 1, 2, \dots, n$
 $v_{\tilde{A}}(\tilde{x}_{ij}) = [x^{(1)}, \tilde{x}_{ij}^{(2)}, \tilde{x}_{ij}^{(3)}] \geq 0$, $\delta_A(z)(\tilde{x}_{ij} = [x^{(1)}, \tilde{x}_{ij}^{(2)}, \tilde{x}_{ij}^{(3)}]) \geq v_{\tilde{A}}(\tilde{x}_{ij} = [x^{(1)}, \tilde{x}_{ij}^{(2)}, \tilde{x}_{ij}^{(3)}])$, $\delta_A(z) [x^{(1)}, \tilde{x}_{ij}^{(2)}, \tilde{x}_{ij}^{(3)}] + v_{\tilde{A}}(\tilde{x}_{ij} = [x^{(1)}, \tilde{x}_{ij}^{(2)}, \tilde{x}_{ij}^{(3)}]) \leq 1$. The given fuzzy transportation is called balanced TP if $\sum_{i=1}^m I a_i = \sum_{j=1}^n I b_j$ if the total fuzzy available is equal to fuzzy requirement.

3.1. Theorems of an Intuitionistic Fuzzy transportation Problem

The solution of an intuitionistic fuzzy transportation problem is fundamentally based on the following two theorems.

3.1.1 Theorem: In an intuitionistic fuzzy transportation problem, if an intuitionistic fuzzy number to every element of the intuitionistic fuzzy cost matrix $\tilde{C}_{ij} = [\tilde{C}_{ij}^{(1)}, \tilde{C}_{ij}^{(2)}, \tilde{C}_{ij}^{(3)}]$ then an transportation that

minimizes the total intuitionistic fuzzy cost on one matrix also minimizes the total intuitionistic fuzzy cost on other matrix. In other words if $\widetilde{x}_{ij} = [x^{(1)}, \widetilde{x}_{ij}^{(2)}, \widetilde{x}_{ij}^{(3)}] = \widetilde{x}_{ij}^* = [x^{(1)}, \widetilde{x}_{ij}^{(2)}, \widetilde{x}_{ij}^{(3)}]^*$, minimizes $\delta_A(zI) = \sum_{i=1}^m \sum_{j=1}^n I C_{ij} \widetilde{x}_{ij}$ and $\sum_{i=1}^m I a_i = \sum_{j=1}^n I b_j$, $I \widetilde{C}_{ij} = I C_{ij} - I u_i - I v_j$ and $i, j = 1, 2, \dots, m, n$ and u_i, v_j are real triangular intuitionistic fuzzy numbers, $I \widetilde{C}_{ij} = u_i + v_j$, select $(+ \emptyset, - \emptyset)$ loop, and then choose $- \emptyset$ position $= \min(I \widetilde{C}_{ij})$, add $\min(I \widetilde{C}_{ij})$ in $+ \emptyset$ position, then subtract $\min(I \widetilde{C}_{ij})$ in $- \emptyset$ position, next to find optimal $I \widetilde{D}_{ij}$ fuzzy solution.

Proof: minimizes $\delta_A(zI) = \sum_{i=1}^m \sum_{j=1}^n I C_{ij} \widetilde{x}_{ij}$
 minimize $\delta_A(zI) = \sum_{i=1}^m \sum_{j=1}^n (I \widetilde{C}_{ij} - I u_i - I v_j) \widetilde{x}_{ij} = \sum_{i=1}^m \sum_{j=1}^n I C_{ij} \widetilde{x}_{ij} - \sum_{i=1}^m \sum_{j=1}^n I u_i - \sum_{i=1}^m \sum_{j=1}^n I v_j$
 $= zI - \sum_{i=1}^m \sum_{j=1}^n I u_i - \sum_{i=1}^m \sum_{j=1}^n I v_j$

This shows that the minimization of the new objective function $\delta_A(zI)$ yields the same solution as the minimization of original objective function zI

3.2. Theorem: In an intuitionistic fuzzy transportation problem with cost $(c_{ij})^I$, if all $(c_{ij})^I \in (A, B)^I$ if all $[x_{ij}] \geq 0$ then a feasible solution which satisfies $\delta_A(zI) = \sum_{i=1}^m \sum_{j=1}^n I C_{ij} \widetilde{x}_{ij} = (A, B)^I$ is optimal for the problem.

Proof: Since all $(I \widetilde{C}_{ij}) \geq 0$ and $\widetilde{x}_{ij}^* \geq 0$, then the current objective of optimal $I \widetilde{D}_{ij} = \widetilde{\delta}_A(zI)$ fuzzy solution can not be negative. The minimum possible value can reach (zI) attain 0^I .

Thus any feasible solution can satisfies \widetilde{x}_{ij}^* that $\sum_{i=1}^m \sum_{j=1}^n I C_{ij} \widetilde{x}_{ij} = (A, B)^I$ will be an optimal.

4. COMPUTATIONAL PROCEDURE FOR INTUITIONISTIC FUZZY TRANSPORTATION PROBLEM

Step 1. In the given intuitionistic fuzzy cost transportation table find the cost value by using ranking procedure as mentioned in section II.

Step 2. In the reduced intuitionistic fuzzy cost transportation table, to find IBFS of the transportation table by using ranking procedure as mentioned in section II.

Step 3. Make the optimal test for the reduced intuitionistic fuzzy cost transportation table obtained from theorem 1 and 2, and then we solve the transportation problem with crisp values by using the VAM procedure to get the initial solution and then the MODI Method to get the optimal solution and obtain the allotment table.

5. NUMERICAL EXAMPLES

Let us consider an intuitionistic fuzzy transportation table with sets representing 3 destinations and 3 origins IFd =

{ IFd1, IFd2, IFd3, } and IFO = I{ FO1, FO2, FO3 } universal set of TP where FO

$$R(\frac{IF}{3}) = (0*2*5) + (2*5*3) + 2(2*1*3) + 2(0*2*3) / 1728 = 0.0243055$$

$$R(\frac{5}{3}) = (1*6*0) + (6*0*1) + 2(6*2*1) + 2(1*6*1) / 1728 = 0.00694$$

$$R(\frac{2}{3}) = (1*4*8) + (4*8*6) + 2(4*5*6) + 2(1*8*6) / 1728 = 32 + 192 + 240 + 96 = 560 / 1728 = 0.32407$$

to Fd, from origin to destination allocation and the rim requirement satisfied by any one the TP method. Suppose that IFBTP(F, D) and (F, O) over C, where F is a mapping F : D → I^C, gives an approximate description

of intuitionistic fuzzy BTP table. The cost TP table (C_I)_I is given whose element are IFBTPS. The problem is to find the optimal Transportation cost so that the total cost of the optimal transportation is minimum.

	D1	D2	D3
O ₁	(7,21,29)(2,2)	(7,20,57)(3,20)	(
O ₂	((4,12,35)(1,12)	(6,14,28)(3,14,
O ₃	(5,9,22)(2,9,	(10,15,20)(5,1	(4,16,19)(1,16,

Solution: The above intuitionistic fuzzy transportation problem can be formulated in the following mathematical programming form of LPP Min z=(2 3 8)(-1,0,1)x₁₁+(-2 3 8)(3,4,5)x₁₂+(-2 3 8)(8,9,10)x₁₃ +(4 9 16)(2,1,0) x₂₁+(4 8 12)(3,-2,-2)x₂₂ +(2 5 8)(2,4,5)x₂₃+(1 4 7)x₃₁+(2 7 12)((2,3,4) x₃₂+(0 5 10)(11,12,14) x₃₃

Subject to

$$x_{11}+x_{12}+x_{13}=(0\ 2\ 5)$$

$$x_{21}+x_{22}+x_{23}=(1\ 6\ 11)$$

$$x_{31}+x_{32}+x_{33}=(1\ 4\ 8) \forall x_{ij}>0 \quad i=j=1,2,3.$$

$$x_{11}+x_{21}+x_{31}=(1\ 4\ 7)$$

$$x_{12}+x_{22}+x_{32}=(1\ 4\ 7)$$

$$x_{14}+x_{24}+x_{34}=(2\ 4\ 8)$$

$$((a_1 a_2 a_3')$$

$$+(a_2 a_3' a_2')+(2(a_2 a_1' a_2')+2(a_1 a_3 a_2'))/1728$$

C11=(2 3 8) (-1,0,1)	R(C11)=(2 * 3 * 1) + (3 * 1 * 0) + 2(0*(-1)*0)+2(-2*1*0) / 1728 = 6/1728 = 0.003
C12 =(-2 3 8)(3,4,5)	R(C12)=(-2 * 3 * 5) + (3 * 5 * 4) + (2(3 * 3 * 4) + 2(-2 * 5 * 4))/1728 = -30+60+72-80/1728=0.0475

C13 =(-2 3 8)(8,9,10)	R(C13)=(-2*3*10)+(3*9*10)+2(3*8*9)+2(-2*10*9)/1728 =-60 +270+432-360=402/1728 = 0.23263
C21 =(4 9 16) (2,1,0)	R(C21)=(4*9*0)+(9*0*1)+2(9*2*1)+2(4*16*1)/1728=36+128/1728=0.0949
C22 =(4 8 12)(3,-2,-2)	R(C22)=(4*8*-2)+(8*-2*-2)+2(8*3*-2)+2(4*12*-2)/1728=-384/1728=0.22222
C23 =(2 5 8)(2,4,5)	R(C23)=(2*5*2)+(5*5*4)+2(5*2*4)+2(2*8*4)/1728=328/1728=0.1898
C31 =(1 4 7) ((2,3,4)	R(C31)=(1*4*4)+(5*5*4)+2(4*3*4)+2(2*8*3) /1728 =(16+100+48+96)/1728=0.15046
C32 =(2 7 12) (3,6,7)	R(C32)=(2*7*7)+(7*7*6)+(2(7*3*6)+2(2*12*6))/1728=98+294+252+288=932/1728=0.53935
C33 = (0 5 10) (11,12,14)	R(C33)=(0*5*14)+(5*14*12)+(2(5*11*12)+2(0*5*12))/1728=840+1320/1728=2160/1728=1.25

and supplies are

$\bar{a}_1 = (2 \ 5)$ (1,3,5)	$R(\bar{a}_1) = (0*2*5)+(2*5*3) + \frac{1}{2}(\bar{a}_1^2 * 3) + 2(0*5*3)/1728 = 0.0243055$
$\bar{a}_2 = (1 \ 6 \ 11)$ (-2, 1)	$R(\bar{a}_2) = (0.1*6*0)+(6*0*-1) + \frac{1}{2}(\bar{a}_2^2 * -1) + 2(1*6*-1)/1728 = 0.00694$
$\bar{a}_3 = (1 \ 4 \ 8)$ (5,6,8)	$R(\bar{a}_3) = (1*4*8)+(4*8*6) + \frac{1}{2}(\bar{a}_3^2 * 6) + 2(1*8*6)/1728 = 32+192+240+96=560/1728 = 0.32407$

And Demand Are

$\bar{b}_1 = (1 \ 4 \ 7)$ (4,5,6)	$R(\bar{b}_1) = (1*4*6+4*6*5+2(4*\bar{b}_1^2 * 5) + 2(1*7*5))/1728 = 24+120+160+70=374/1728 = 0.2164$
$\bar{b}_2 = (1 \ 4 \ 7)$ (1,2,3)	$R(\bar{b}_2) = (4*3*2+4*3*2+2(4*\bar{b}_2^2 * 2) + 2(1*7*2))/1728 = 12+24+16+28/1728 = 80/1728 = 0.04629$
$\bar{b}_3 = (2 \ 4 \ 8)$ (3,4,5)	$R(\bar{b}_3) = (2*4*5+4*5*4+2(4*\bar{b}_3^2 * 4) + 2(2*8*4))/1728 = 100+80+96+128/1728 = 404/1728 = 0.233769$

Remark 5.1: In the above problem since condition 3.3 (Equation 3.3) is satisfied by all the tri- numbers (cost, supply and demand), for any value of α we will get the same table as below.

Unbalanced transportation \Leftrightarrow total supply \neq total demand, then convert into balanced one, balanced transportation

	D ₁	D ₂	D ₃	Supply
O ₁	0.003	0.0475	0.23263	0.024305
O ₂	0.0949	0.22222	0.1898	0.00694
O ₃	0.15046	0.53935	1.25	0.32407
O ₄	0	0	0	0.144513
deman	0.216	0.0462	0.23376	0.496459

Step 2: Using VAM procedure we obtain the initial solution as

0.003	0.0475	0.23263 0.0243055
0.0949	0.2222	0.1898 0.00694
0.1504 0.2164	0.53935 0.04629	1.25 0.06
0	0	0 0.1445135

Next by using the MODI method we shall improve the solution and get the optimal solution as $u_1=0.23263, u_2=0.1898, u_3=1.25, u_4=0, V_1=-1.09954, V_2=-0.71065, V_3=0, D_{11}=0.86391, D_{12}=0.52552, D_{21}=0.74307, D_{31}=1.09954, D_{32}=0.71065$

Hence all $D_{ij} \geq 0$, which is an unique optimal solution and $z = 0.139485$.

Inference: $I_z < F_z$

Now using the allotment rules, the solution of the problem can be obtained in the form of fuzzy numbers.

Therefore the fuzzy optimal solution for the given transportation problem is

$$x_{13} = (-2 \ 3 \ 8)(8,9,10) \quad x_{23} = (2 \ 5 \ 8)(2,4,5)$$

$$x_{31} = (1 \ 4 \ 7) ((2,3,4) \ x_{32} = (2 \ 7 \ 12) (3,6,7)$$

$$x_{33} = (0 \ 5 \ 10) (11,12,14) \quad x_{43} = (0 \ 0 \ 0) (0 \ 0 \ 0)$$

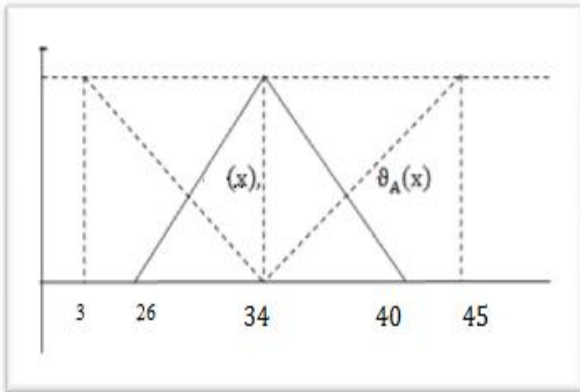
and the fuzzy optimal value of $z = (0.23263+0.1898+0.15046+0.53935+1.25+0) = 3.06224$.

The intuitionistic fuzzy minimum cost is calculated as $(-2 \ 3 \ 8)(8,9,10) + (2 \ 5 \ 8)(2,4,5) + (1 \ 4 \ 7) ((2,3,4) + (2 \ 7 \ 12) (3,6,7) + (0 \ 5 \ 10) (11,12,14) + (0 \ 0 \ 0) (0 \ 0 \ 0) = (3 \ 24 \ 45)(26 \ 34 \ 40)$

In the above example it has been shown that the total optimal cost obtained by our method remains same as that obtained by converting the total intuitionistic fuzzy cost by applying the ranking method [4].

Results and discussion:

The minimum total intuitionistic fuzzy transportation cost is $Z^I = (3 \ 24 \ 45)(26 \ 34 \ 40)$. The result in (1) can be explained (Refer to figure 1) as follows:



- (a) Transportation cost lies in $[26, 40]$.
- (b) 100% experts are in favour that transportation cost is as 3.06224,

$$\delta_A(z) = 3.06224, \nu_{\tilde{A}}(x_{ij}) = 46.93776.$$

- (c) Assuming that $\delta_A(z)$ is a membership value and $\nu_{\tilde{A}}(x_{ij})$ is a non-membership value at x . Then 100% experts are in favour and $100\nu_{\tilde{A}}(x_{ij})$ experts are opposing but $100(1 - \delta_A(z) - \nu_{\tilde{A}}(x_{ij}))$ are in confusion that transportation cost is x . \therefore We accept Intuitionistic fuzzy value. There is no significant difference between before and after Transportation cost.

$\delta_A(z) = \begin{cases} 0 & \text{for } z < 26, \\ \frac{z-26}{8} & \text{for } 26 \leq z \leq 34, \\ 1 & \text{for } z = 34 \\ \frac{40-z}{6} & \text{for } 34 \leq z \leq 40 \\ 0 & \text{for } z > 40 \text{ otherwise} \end{cases}$ <p>Where $26 \leq 34 \leq 40$</p>	$\nu_{\tilde{A}}(x) = \begin{cases} 1 & \text{for } z < 34, \\ \frac{34-z}{31} & \text{for } 34 \leq z \leq 40 \\ 0 & \text{for } z = 40 \\ \frac{x-34}{31} & \text{for } 34 \leq z \leq 45 \\ 1 & \text{for } z > 45 \text{ otherwise} \end{cases}$ <p>Where $a_1' \leq a_1 \leq a_2 \leq a_3 \leq a_3'$ and $\delta_A(z), \nu_{\tilde{A}}(z) \leq 0.5$ and $\delta_A(z) + \nu_{\tilde{A}}(z) \leq 1$ for all $z \in R$.</p>
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Values of $\delta_A(z)$ and $\nu_{\tilde{A}}(z)$ at different values of x can be determined using equations given below.

CONCLUSION

In this paper, we discussed finding a solution of Intuitionistic fuzzy transportation problem in which cost coefficients are triangular intuitionistic fuzzy numbers. The total optimal cost obtained by our method remains same as that obtained by converting the total ranking fuzzy cost by applying the ranking method [2]. Also the membership values of the fuzzy costs are derived. This technique can also be used in solving other types of problems like, project schedules, transportation problems and network flow problems.

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