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# LEADING ORDER CALCULATION OF POLARIZED STRUCTURE FUNCTIONS AND PROTON ASYMMETRY BY THE METHOD OF STATISTICAL APPROACH

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## Abstract:-

We evaluate the proton Asymmetry $A_1^p(x)$in the kinematic region 0.1 < x < 0.7 for $Q^2$ =1Gev$^2$ using Thermodynamical bag Model(TBM). The theoretical prediction is compared with recent experimental results of COMPASS and SMC. Ourresults are in good agreement with experimental results. Theoretical evaluation of first moment $g_1^p$at $Q^2$ = 3 GeV$^2$ is consistent with experimental results.

**Keywords: -** DIS, Structure function, Asymmetry, TBM

## 1. INTRODUCTION

Quantum Chromo Dynamics(QCD) theory is mostly accepted for describing the strong interactions[1]. Ever since the quark-spin contribution for the spin of the proton was determined by EMC and it was not fulfilled the all determinations. The polarized Deep Inelastic Scattering(DIS) of nucleon experiment is an important tool to investigate the inner structure of the nucleon. The improved accuracy of data collected by experiments at CERN and SLAC in past few years has motivated and allowed perturbative Quantum Chromo Dynamics(pQCD) analyses of the nucleon spin dependent structure functions at NLO[2-3]. The spin structure of the nucleon is composed by intrinsic quark spin, orbital angular momentum of quarks and gluon spin.Mathur[4] estimated the quark orbital angular momentum of the nucleon on the basis of lattice QCD approach. From this calculation, the total angular momentum of quarks carried60% of the proton spin and 40% proton spin is originated by the gluon orbital angular momentum. In naive quark model, three valence quarks provide the quantum numbers of proton which gives the sum of the quark spins should be equal to the proton spin. It is observed from Ellis-Jaffe integral that the sum of helicities much smaller than the value of half which gives to rise the proton "spin puzzle".

The spin dependent structure in deep inelastic lepton nucleon scattering is important tool for untravelling the quark and gluon spin distribution inside a nucleon. From the past decade the structure function is focused on kinematic dependence to study the polarized quark and gluon distribution function. In recent years, the polarized deep inelastic scattering experiments demonstrated by COMPASS[5-6] and SMC[7] for the investigation of spin structure and quark distribution function inside the nucleon. At finite $Q^2$ the structure function is violated due to the gluon radiation by both initial and scattered quarks. These gluon radiative corrections cause a logarithmic $Q^2$ dependence to the structure functions which has been verified by experimental data[8] and can be precisely evaluated in pQCD using the DGLAP evaluation equations[9].

The Polarizedparton distribution functions(PDFs)behavior is predicted as $x \rightarrow 1$ offered by severalstatistical models. Bourrely[10] evaluated the large x behavior of polarized PDFs inwhichthe nucleontreated as a gasof massless constituent at thermal equilibrium using chirality and DIS data to constrain the thermodynamical potential of each constituents. In leading order perturbative QCD(pQCD), the valence quark orbital angular momentum is assumed to be ignored at large x and $Q^2$, thus leading to hadron helicity conservation[11]. The investigation of polarized PDFs at large x region is one of the importantgoal persued by ongoing in JLAB experiments[12]. The several experimental data for proton and neutron asymmetries in inclusive DISavailablefromJLAB[13-15] experiments. In the present our thermodynamical model, we evaluate proton spin dependent structure function and asymmetry in large x scale for $Q^2$>1GeV$^2$ and evaluated results compared with the COMPASS[5-6] and SMC[7].

## Thermodynamical Bag Model:

Thermodynamical Bag Model (TBM) first developed by Ganesamurthy et.al[16-20] considering the nucleon to be in the Infinite Momentum Frame (IMF), where the quarks and gluons are treated as fermions and bosons respectively. The invariant mass(W) of the final hadron and the equation of states are

$$\left[\varepsilon(T)V + BV\right]^2 = W^2 = M^2 + 2M\nu - Q^2 \quad (1)$$

$$6(n_u - n_{\bar{u}}) = \frac{2}{V} = \mu_u T^2 + \frac{\mu_u^3}{\pi^2} \quad (2)$$

$$6(n_d - n_{\bar{d}}) = \frac{1}{V} = \mu_d T^2 + \frac{\mu_d^3}{\pi^2} \quad (3)$$

Where ε(T) is the energy density of the system at a temperature T, V is the volume of bag, B is the bag constant, W is the invariant mass of excited nucleon at T, $\nu$ is the energy transfer, $Q^2$ is square of four momentum transfer, M is the mass of the nucleon at ground state, $6(n_u - n_{\bar{u}})$ is number density of up quark, $6(n_d - n_{\bar{d}})$ is the number density of down quark, $\mu_u$is the chemical potential of up quark and $\mu_d$ is the chemical potential of down quark.

The total energy density ε(T) of the bag can be written by the sum of energy densities of up quark, down quark and gluon is given by

$$\varepsilon_u + \varepsilon_{\bar{u}} = \left(\frac{1}{8\pi^2}\right)\mu_u^4 + \left(\frac{1}{4}\right)\mu_u^2 T^2 + \left(\frac{7\pi^2}{120}\right)T^4 \quad (4)$$

$$\varepsilon_d + \varepsilon_{\bar{d}} = \left(\frac{1}{8\pi^2}\right)\mu_d^4 + \left(\frac{1}{4}\right)\mu_d^2 T^2 + \left(\frac{7\pi^2}{120}\right)T^4 \quad (5)$$

$$\varepsilon_g = \frac{\pi^2 T^4}{30} \quad (6)$$

$$\varepsilon(T) = d_q(\varepsilon_u + \varepsilon_{\bar{u}}) + d_q(\varepsilon_d + \varepsilon_{\bar{d}}) + d_g \varepsilon_g \quad (7)$$

Where $d_q$= 6 and $d_g$=16 denotes the degeneracy of quarks and gluon orderly.

The statistical Parton Distribution Functions are expressed as

$$q_i(x,Q^2) = \left(\frac{6V}{4\pi^2}\right)M^2 T x \ln\left\{1+\exp\left[\left(\frac{1}{T}\right)\left(\mu_i - \frac{Mx}{T}\right)\right]\right\} \quad (8)$$

$$\bar{q}_i(x,Q^2) = \left(\frac{6V}{4\pi^2}\right)M^2 T x \ln\left\{1+\exp\left[\left(\frac{1}{T}\right)\left(-\mu_i - \frac{Mx}{T}\right)\right]\right\} \quad (9)$$

$\mu_i$ is the chemical potential of quark with the flavour 'i'. Here 'i' denotes u or d quark.In order to relate the PDF's with $\Lambda_{QCD}$, which is quark gluon coupling parameter, we introduce the strong quark gluon coupling constant. The experimental fit could be made by considering only with the QCD corrections. The quark and anti-quark distributions are modified by including QCD parameters as,

$$q_i'(x,Q^2) = q_i(x,Q^2)\left(1 - \frac{\alpha_s(Q^2)}{2\pi}\right) \quad (10)$$

$$\bar{q}_i{}'(x,Q^2) = \bar{q}_i(x,Q^2)\left(1-\frac{\alpha_s(Q^2)}{2\pi}\right) \quad (11)$$

The strong running coupling constant ($\alpha_s$) for various $Q^2$ is evaluated using the Next to Leading Order (NLO) solution.

$$\alpha_s(Q^2)=\frac{4\pi}{\beta_0\ln(Q^2/\Lambda^2)}\left[1-\frac{\beta_1}{\beta_0}\frac{\ln(\ln(Q^2/\Lambda^2))}{\ln(Q^2/\Lambda^2)}\right] \quad (12)$$

Where $\beta_0=11-(2N_f/3)$ and $\beta_1=102-(38N_f/3)$. In order to account for heavy quark threshold correction and target mass effect together, a substitution of x is made with $\xi$ .

$$\xi = \frac{2x((1+m_s^2)/Q^2)}{1+\sqrt{1+(4M^2x^2/Q^2)((1+m_s^2)/Q^2)}} \quad (13)$$

$m_s$is the mass of the strange quark. Here we assume strange quark mass as 100MeV and $\Lambda_{QCD}$=350MeV.

## Theoretical estimation of proton asymmetry:

The structure function $F_1$ and $F_2$ are related by Callon-Gross relation

$$2xF_1(x) = F_2(x) = \sum_i e_i^2 x q_i(x) \quad (14)$$

In this relation, structure functions depend only on x. This means that the lepton scatters on particles which do not involve any scale i.e. on point-like particles. The fact that the structure function indeed do not depend on $Q^2$, the so called scattering discovered at SLAC[21] was the experimental validation of the parton model. The unpolarized structure function of proton and neutron are evaluated with the inclusion of up and down anti-quarks,

$$F_2^p = x\left[\frac{4}{9}(u'(x)+\bar{u}'(x))+\frac{1}{9}(d'(x)+\bar{d}'(x)\right] (15)$$

$$F_2^n = x\left[\frac{4}{9}(d'(x)+\bar{d}'(x))+\frac{1}{9}(u'(x)+\bar{u}'(x)\right] (16)$$

The structure function $g_1(x)$ is interpreted as the difference between two probabilities $q_i^{\uparrow}(x)$ and $q_i^{\downarrow}(x)$ averaged over the quark flavor charges.

$$g_1(x) = \frac{1}{2}\sum_i e_i^2 \Delta q_i(x) \quad (17)$$

Where $\Delta q_i(x) = \left[q_i^{\uparrow}+\bar{q}_i^{\uparrow}\right]-\left[q_i^{\downarrow}+\bar{q}_i^{\downarrow}\right]$. It measuresthe electric charge weighted difference between quarks with spins parallel and antiparallel to the nucleon spin. Hence spin dependent structure function of proton and neutron are given by

$$g_1^p = 0.5\left[\frac{4}{9}\Delta u'(x)+\frac{1}{9}\Delta d'(x)\right] \quad (18)$$

$$g_1^n = 0.5\left[\frac{4}{9}\Delta d'(x)+\frac{1}{9}\Delta u'(x)\right] \quad (19)$$

Where $\Delta u'(x), \Delta d'(x)$are the spin distribution function of up and down quark with anti-quarks given by

$$\Delta u'(x)=\left[(u'(x)+\bar{u}'(x))-\frac{2}{3}(d'(x)+\bar{d}'(x)\right]\cos 2\theta(x)$$

(20)

$$\Delta d'(x)=\left[-\frac{1}{3}(d'(x)+\bar{d}'(x)\right]\cos 2\theta(x) \quad (21)$$

Where

$$\cos 2\theta(x) = \frac{1}{1+\left(\frac{H_0}{\sqrt{x}}(1-x)^2\right)} \quad (22)$$

isknown as the spin dilution factor[22]. Since the spin dilution factor is not derived from first principles it is not adjusted to satisfy the Bjorken sum rule which is considered as the fundamental test of QCD. This enables to determine the valence quark distribution explicity. Here $H_0$ is a free parameter chosen 0.09 to satisfy the Bjorken sum rule.The contribution of polarized valence quark distribution and strange quark distribution are evaluated at $Q^2$= 10 $GeV^2$which are consistent with COMPASS results[23] and it is given in the table I.

| | COMPASS | TBM |
|---|---|---|
| $\Delta u_v + \Delta d_v$ | 0.41±0.07±0.06 | 0.519 |
| $\Delta s + \Delta\bar{s}$ | - 0.09±0.01±0.02 | -0.074 |

**Table: 1**

Proton asymmetry is expressed by the ratio between spin dependent structure function and unpolarized structure function of proton. Since $g_1$ and $F_1$ are evaluated at same $Q^2$ in leading order QCD, $A_1$ is expected to vary slowly with $Q^2$.

$$A_1^p(x,Q^2) = \frac{g_1^p(x,Q^2)}{F_1^p(x,Q^2)} \qquad (23)$$

Non relativistic quark modelpredicted the proton asymmetry $A_1^p$ = 5/9 as $x \to 1$ on the basis of SU(6) symmetry. $A_1$ is more positive at large x due to positive polarization of up and down quarks. In perturbative QCD, $A_1$ is expected to unity as $x \to 1$. In this kinematic region, the contribution of both sea and gluon are small and we study the contribution of valence quarks and their orbital angular momentum to the nucleon spin. Relativistic constituent quark model is slightly overestimates the proton asymmetry. In the present work, the valence quarks are dominated at large x region and asymmetry of proton is expected to unity as $x \to 1$.

## Results and discussions:

The evaluated results of spin dependent structure function $g_1^p(x)$ and proton asymmetry $A_1^p$ using TBM based quark distribution function isshown in figure I and II respectively and the results are compared with COMPASS[5-6]andSMC[7].

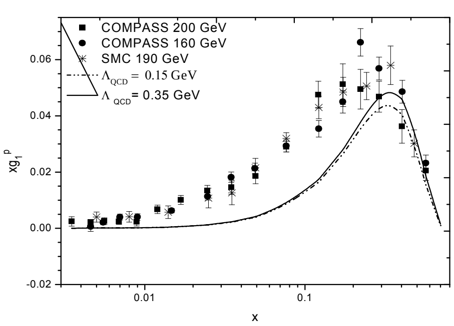


**Figure 1:TBM results(solid line) atQ$^2$ = 1 GeV$^2$is compared withCOMPASS data 200GeV (squares), COMPASS 160GeV (circles) and SMC 190 GeV (stars).**

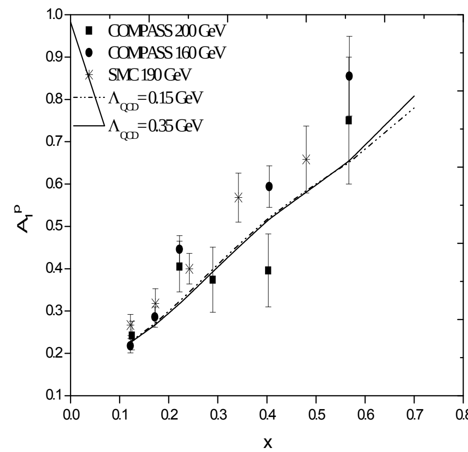


**Figure 2: TBMresults(solid line) at Q$^2$ = 1 GeV$^2$is compared with COMPASS data 200 GeV (squares), COMPASS 160GeV (circles) and SMC 190 GeV (stars) results.**

In the present work, we have chosen the large kinematic region and the valence quarks are more dominated in this region and the ratio of structure functions can be evaluated through the interaction between quarks.The spin dependent structure function is employed in our estimation is used to derive the spin asymmetry of proton. Thespin dependent structure function increases with increasing Bjorken variable and reach maximum value at x=0.341. Further, increasing x value, the polarized structure function $xg_1^p$ decreases. In this whole region, up quark spin distribution is positive and down quark spin distribution is negative. The variation of polarized structure function with Bjorken variable due to the contribution of up and down quark distribution functions areanalysed. The deviation of $xg_1^p$ upto x=0.341 compared with experimental data is due to the dominance of sea quarks which is natural consequence of this model at low x values.The proton asymmetry increases with increasing Bjorken variable x. This is due to the up quark spin distribution attains positive asymmetry and down quark spin distribution attains negative asymmetry. The proton asymmetry becomes positive in the whole evaluated region because of up quarks are dominated over down quarks. The evaluation of first moment of polarized structure function$g_1^p(x)$usingThermodynamocal bag modelin which $g_1^p = 0.128$for$Q^2 = 3$ GeV$^2$. These results are in good agreement with COMPASS[6] results. Furthermore the present theoretical model which reduces the uncertainties of parton distribution functions(PDFs) both small and large x region compared to experimental observations, we may get better improvement knowledge of the nucleon structure function.

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